Kuikeg's pdf on the four bearing method (and its building blocks) is an excellent read for anyone interested in sub warfare, but I feel like it skips over a few steps and details that could help to really understand why the method works, therefore making it more digestible and easier to remember in combat. I didn't fully get it until I started drawing some examples myself, and I decided to rewrite his notes to hopefully make the method more accessible to newcomers, at the cost of sacrificing quite a bit of the short and to the point format of his. I will also try to give a little more context and breadth to the topic, by including a few more use cases, and my take on some tactical aspects of these methods. I hope that by the end of your reading, you will be able to go back to Kuikeg's original document and use it as a quick reference, armed with a better understanding of the concepts. Writing this down definitely helped me understand this much better, so I hope it will be of some use to you as well.

Obvious disclaimer, I am by no means an expert on subsims, and my geometry courses are a distant memory, so this is more of an excuse for myself to understand all of this, and everything here is very much open to suggestions and input to make it a good resource for past and future players!

And for those of you wondering, I've been using an excellent online tool by Math10.com for all my drawings.

- Neuro

Why bother with bearings at all

When within visual range, two stadimeter readings will do the trick, so why bother with geometry? After all, you can "chase" a hydrophone contact until visual range and then plot your next move. This is absolutely true, but there are situations where PD is not advisable. Maybe you are lurking off a port and trying to position yourself to catch an outbound convoy, if you simply set pure pursuit to "chase the bearing" you will waste time and fuel, and increase your exposure to patrols. Or maybe you are hunting on a foggy night, with barely enough visibility to make a last second fix well within torpedo range, so positioning yourself optimally is the only chance to get the most out of a narrow shooting window. In these cases, you can get a very good read on your target and set up your intercept early on, and keep refining your solution as you move closer to your attack point. And finally, because it's fun and gives you one more tool to master.
Whatever the reason, we are studying a situation in which your only source of intel on a target is a passive, bearing only sensor (hydrophone, periscope at long distance, radio direction finder etc).
There are obviously some limitations to these methods (limited bearing resolution of hydrophones, and a high dependence on your plotting accuracy), but however you look at it, they will only work if a few assumptions are met. Generally speaking, your solution will only be usable as long as your target is maintaining constant speed and course. However, even for a zig-zagging convoy, as the average course should be more or less constant, you will still get a pretty good idea of where they are going if you monitor the convoy long enough. The good news is that with practice, it should be possible to get an usable solution rather quickly and from a very respectable distance, and even if you are on the move.

First stop: the static "three bearings" (plus one) method

Our first example is a simple case, where you can afford the luxury of sitting in one spot long enough (hence "static" in the title). How long is required depends on the situation, but we will get to that. In general, if you are sitting still, you can determine the course of a contact with as little as three bearings, when the time between those readings is equal. We will then add a further step that lets us calculate the exact speed and distance.

So let's suppose we dive for a hydrophone sweep and hear the faint sound of a merchant somewhere. We stop the engines (or even order reverse for a little while to zero our speed quicker), and when we are dead in the water (or close enough!), we take the first bearing and note the time.
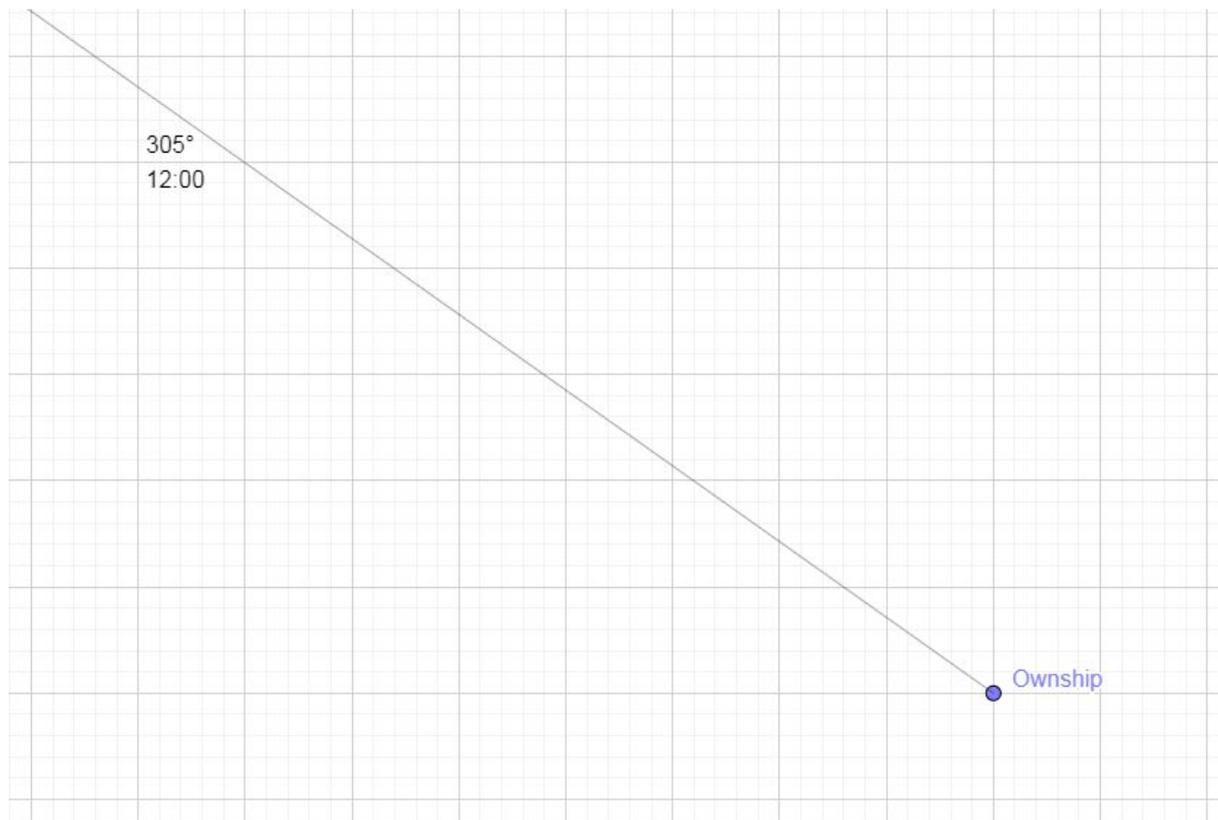
We are now sitting still, and listening. All we know is something is out there; depending on what game you're playing, you might be able to tell if they are closing or opening distance, but for now that's irrelevant. Now we wait; we want a change in bearing of at least a few degrees, or our solution will not be very precise. There is of course a chance that the bearing rate of change will be zero, if the target is moving directly towards us (or directly away). In such case, use your judgment (and your ears, if the sound is getting louder - great!, if not, it might be time to surface and give chase).

But for now, let's suppose we are seeing a positive change in bearing, and that after five minutes, our target is on a bearing of 311°. You might also want to wait a little longer to give

you more time to do the plotting. Once again, we mark the bearing using our in game tools, as well as the time (on the map or on a piece of paper).
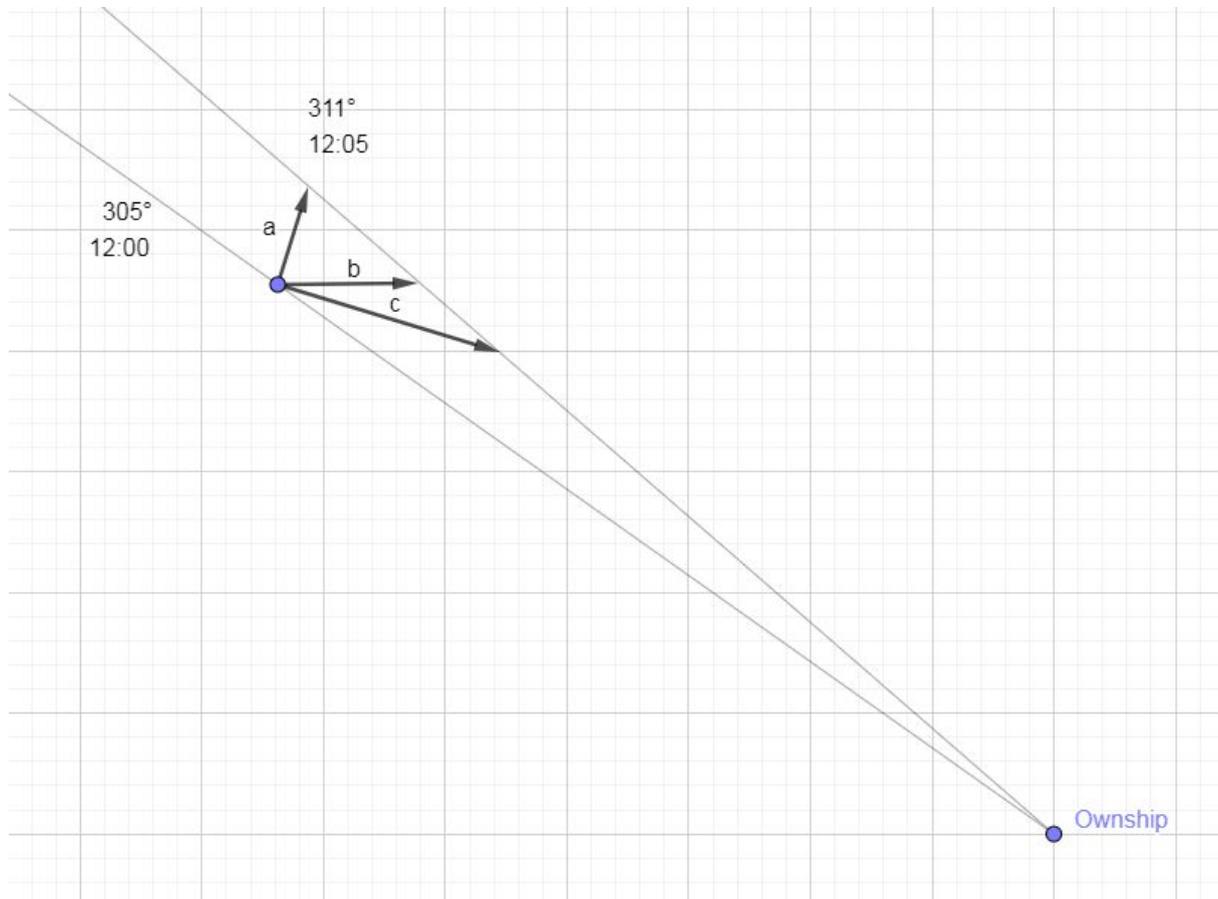

We now know that the target was on 305° five minutes ago, and at 311° now; this only gives us a rough sense of where they might be heading, but as shown in the picture above, there is still an infinite combination of ranges, speeds, and bearings that match our information (a, b, and c are only examples of possible solutions with our current information, and we don't need to plot them in practice).

This is not enough, so we sit and wait five more minutes (same as our first time interval). At 12:10, we get a new bearing, and plot it on the map:
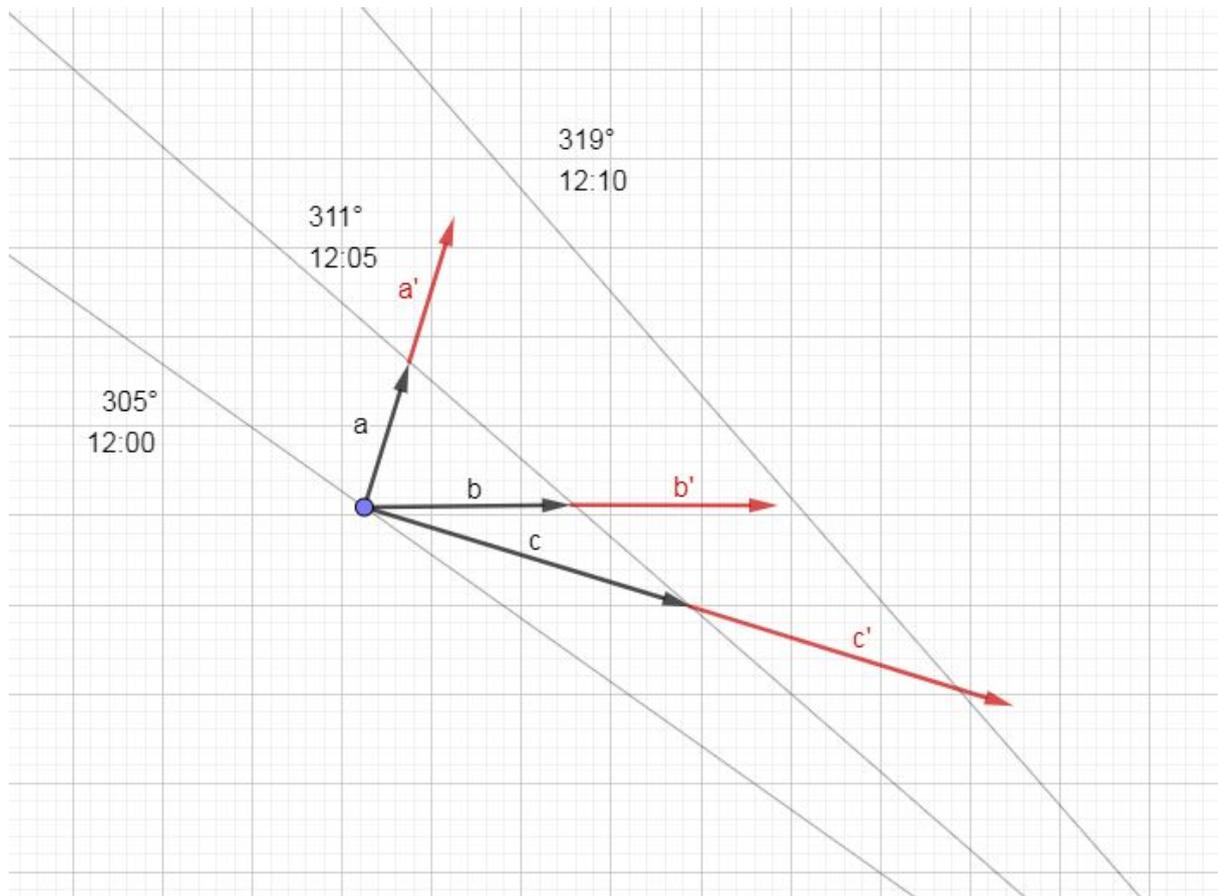
As you can see, the third bearing starts clearing things up for us. The three red vectors show that our previous example "guesses" do not match our new information; the third bearing helps us weed out a lot of incorrect solutions, and as we will see in a moment, is enough to restrict our options to a smaller infinity: we have now all the elements to determine our target's course (but not range, or speed). Notice, that a and a' (and the other pairs of vectors as well) have the same length, as they represent distances covered by our hypothetical ship over two equal intervals of time. This will help explain the next step.

Before we move on, it's time to rev up the engines and move away from our current position! Do this as soon as you have jotted down the third bearing, as we want to move as far as possible from the starting point in the next five minutes. Where to, you might ask? Well, the optimal heading for our purposes would be at a right angle to the next bearing we will take (and we will predict it), but since we want to get moving as soon as we can and we already have a rough idea of the target heading, we'll take a heading perpendicular to the third bearing, so that we don't lag behind too much (so in our case, 319°+90° = 49°). If the time interval you chose allows for it, you can surface the boat to cover more distance, but I wouldn't advise it if you have less than five minutes or so, just to make sure you don't miss your window while diving back down to PD.
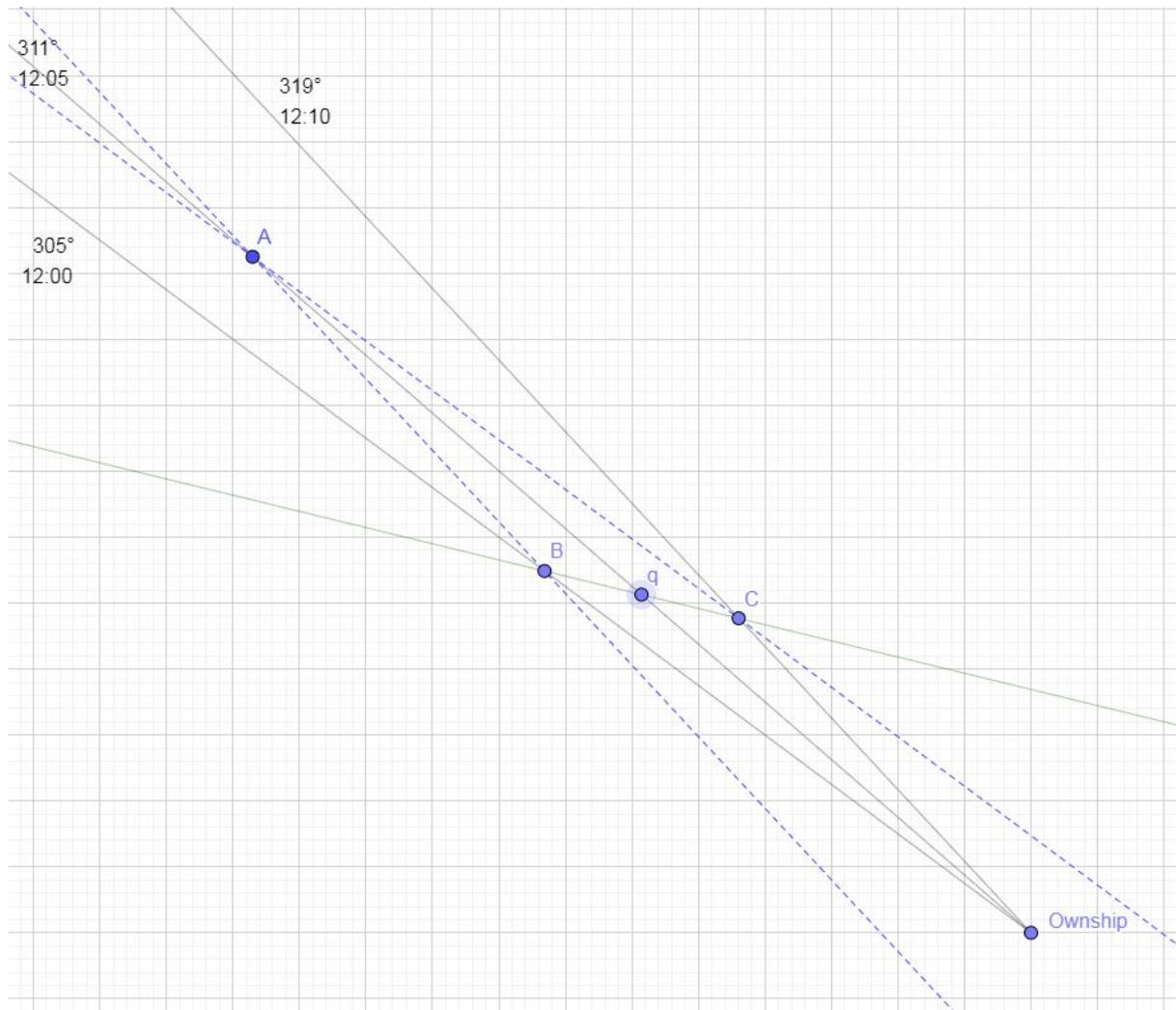
Back to the map, now we have to figure out target course. We first choose a point anywhere on our second bearing (12:05), called A in the figure. Then, we draw two lines, parallel to our first and third bearings, through point A (the two blue dashed lines). They will intersect bearings 1 and 2 in the two points B and C. Now all that's left is to join B and C with one more line (green in the pictures), and we got our target's course (obviously, since our first bearing was to the west, the target is moving west to east, so no ambiguity there).

Note that this is because we constructed a parallelogram defined by Ownship, C, A, and B, and since the diagonals in parallelograms always bisect each other, this means that the segments Bq and qC are of equal lengths.

In other words, we made a geometrical construction that answers the following question: *What course guarantees that a ship, starting at any distance from ownship on bearing 1, and traveling at a constant speed, would end up on bearing 2 after five minutes, and on bearing 3 after five more minutes?*

This might sound unnecessary, but I think it's crucial to understanding why we do what we do, and having a good grasp of the geometrical approach will help understanding the more complicated case further down the road.

Now that we know the course, how do we determine range and speed? Well, remember we started moving away from our starting position. While we're traveling (and hopefully well before the next five minute mark), let's find one last line on our plot: the predicted fourth bearing.
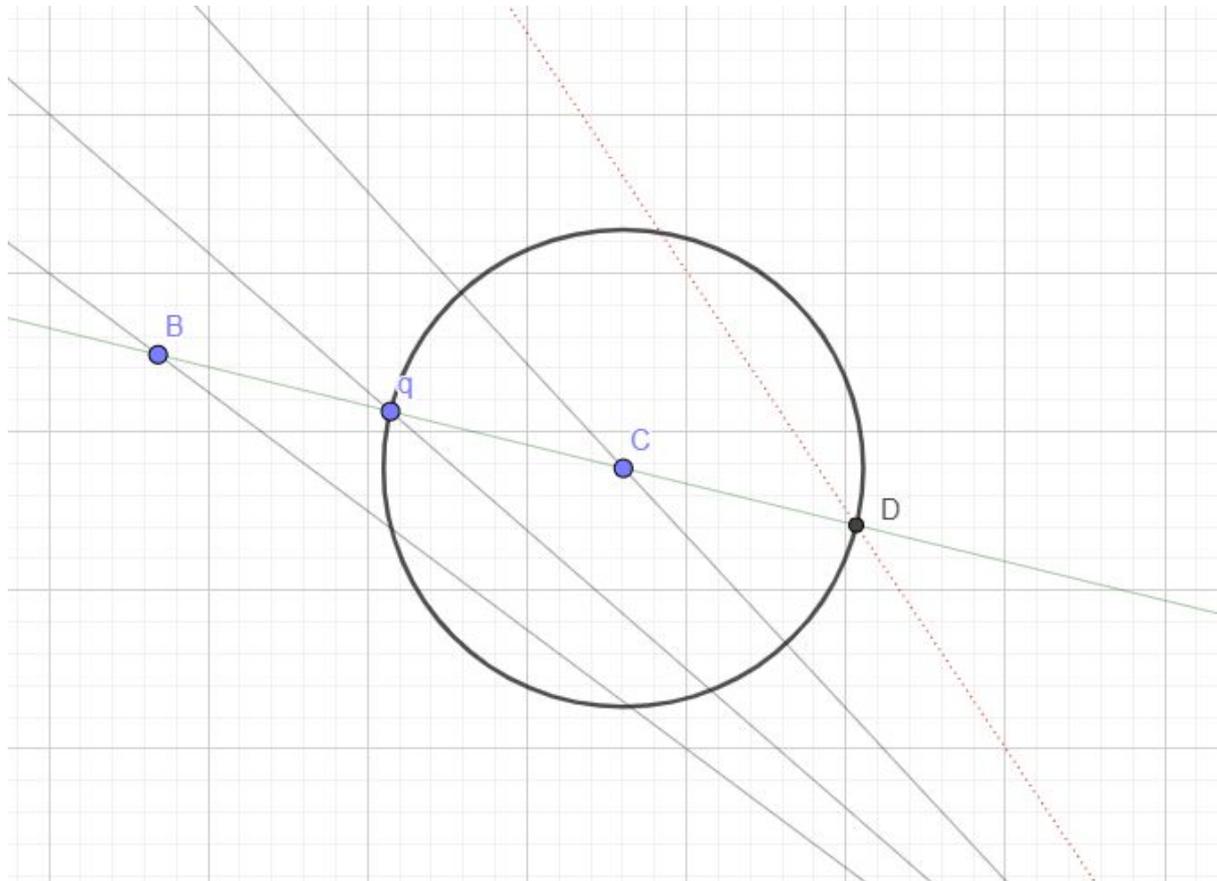
We draw a circle centered on C (from the previous plot), and with radius qC; this gives us point D, the intersection of our circle and the green course line we found earlier. This simply means that now that we know the course, we are just predicting the next bearing that we would be logging, if we stayed at the starting position, after five more minutes (the red line). We still don't know the distance to target, but if we did things right and started moving away from our starting position, after five minutes we can log one more bearing…
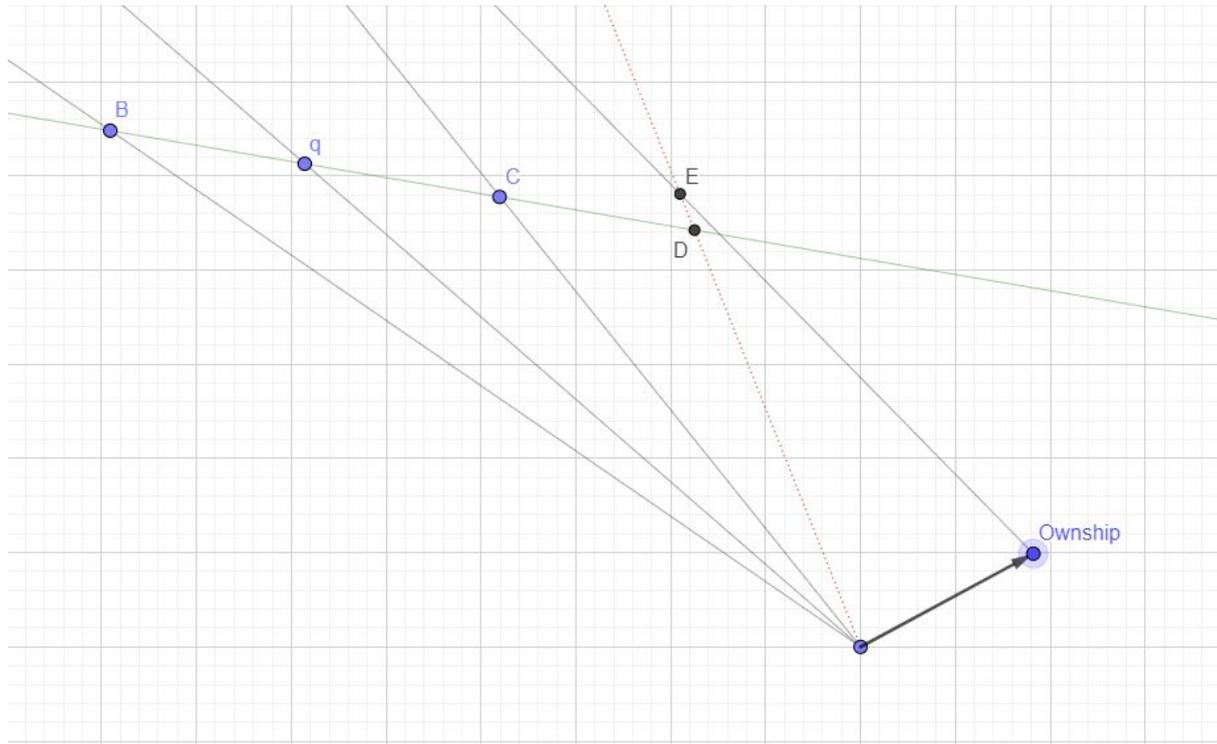
The last bearing we found after moving intersects the red line in E, which, if we have been accurate enough, will be the target's exact position at 12:15. If you don't believe me, think about it another way; all the work so far was really to draw the red line (or predicted bearing), and use it together with a new bearing to triangulate the target's position. By sitting still for the first three readings, we have mathematical certainty that the target must be somewhere on the red line at 12.15 (at least accurate to the extent of our plotting, and our sonarman's ability to provide an accurate bearing), so if at the same time we take his bearing from another point, we have his position. In principle this is the same result you would get if you triangulated with a friend some kilometers away giving you his bearing to target, with the added benefit that we already have everything we need to get speed and course as well (and we don't need to coordinate over the radio).
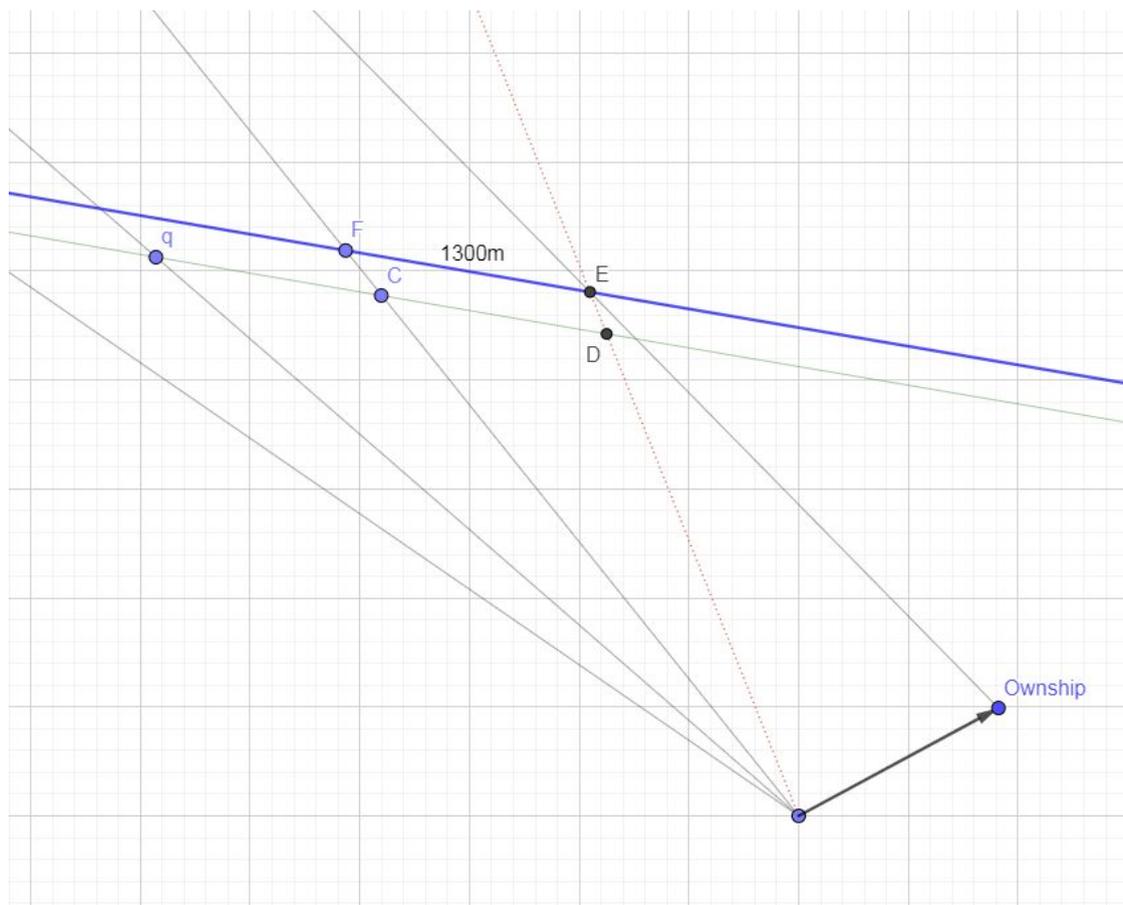
Now all that's left to do is draw a line through E (in blue), parallel to our previously determined course (the green line); the blue line now is the actual, physical track of the target, and by measuring the length of EF, which then has no other choice than to represent the actual distance covered by our target over the last five minutes, we can determine target speed. You can use tables, a calculator, or your head, but either way, in our case we have found that the target is currently northwest, at approximately 3km, with a heading of 100°, and a speed of about 8.5 knots. What happens next is usually some sort of intercept plotting to put you ahead of the target so that you can ambush them while submerged and hit them with that perfect 90° shot, but for the moment, it's out of the scope of this (not so short…) document.

Now let's move on to something a little more complicated!

Alternative bearing method

Next up, we will look at an alternative geometrical approach to determining target course from three bearings from a static position. This doesn't really give us much more than the previous, but as we will see later, it is used as the last step of the "real" four bearings method, which can be used on the move, so you might as well practice it!

The setup is the same as the previous example, so we will move directly to plotting course from three collected bearings. We start by taking two arbitrary points on bearing 1 and 2:
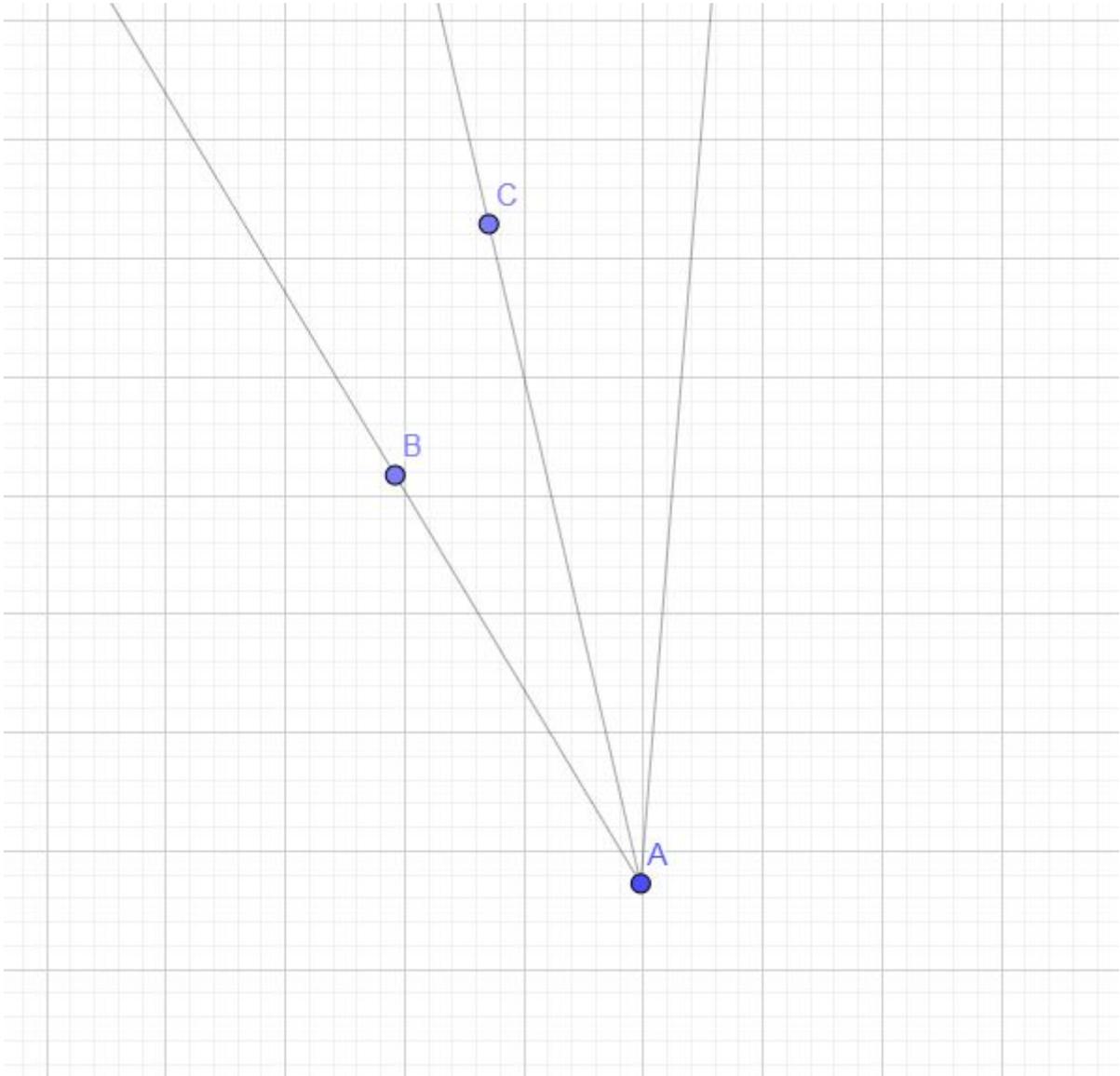
Then, by drawing a circle centered on C and with radius BC, we find point D, on the opposite side of the circle.
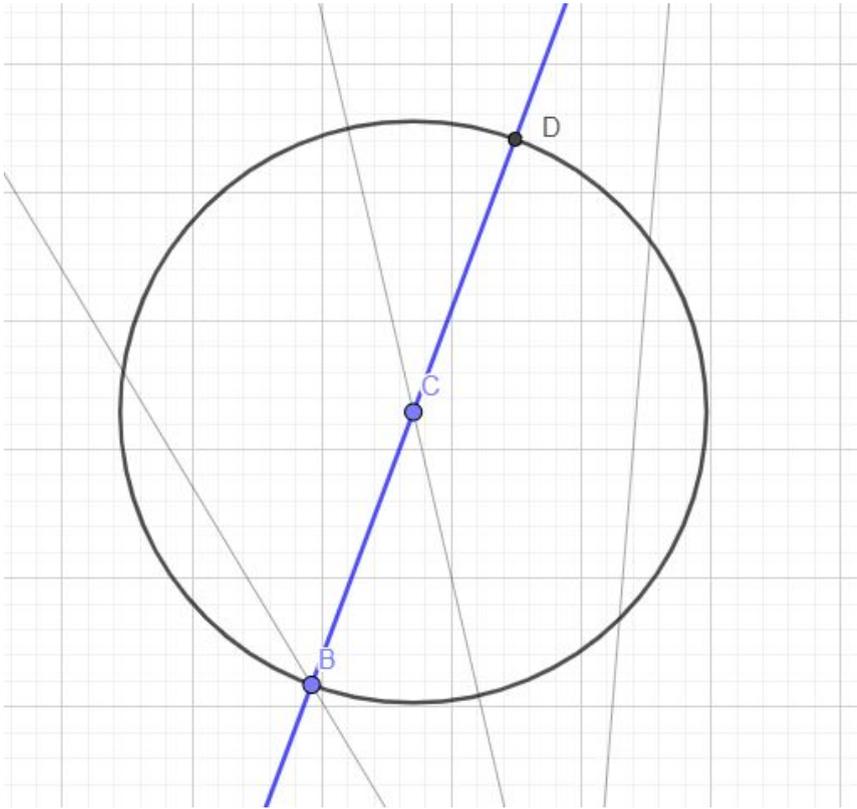
Now all we need to do is draw a line parallel to bearing 2, going through point D. This gives us point E, and by drawing a line through B (our first point) and E, the usual green line, we have our target's course.
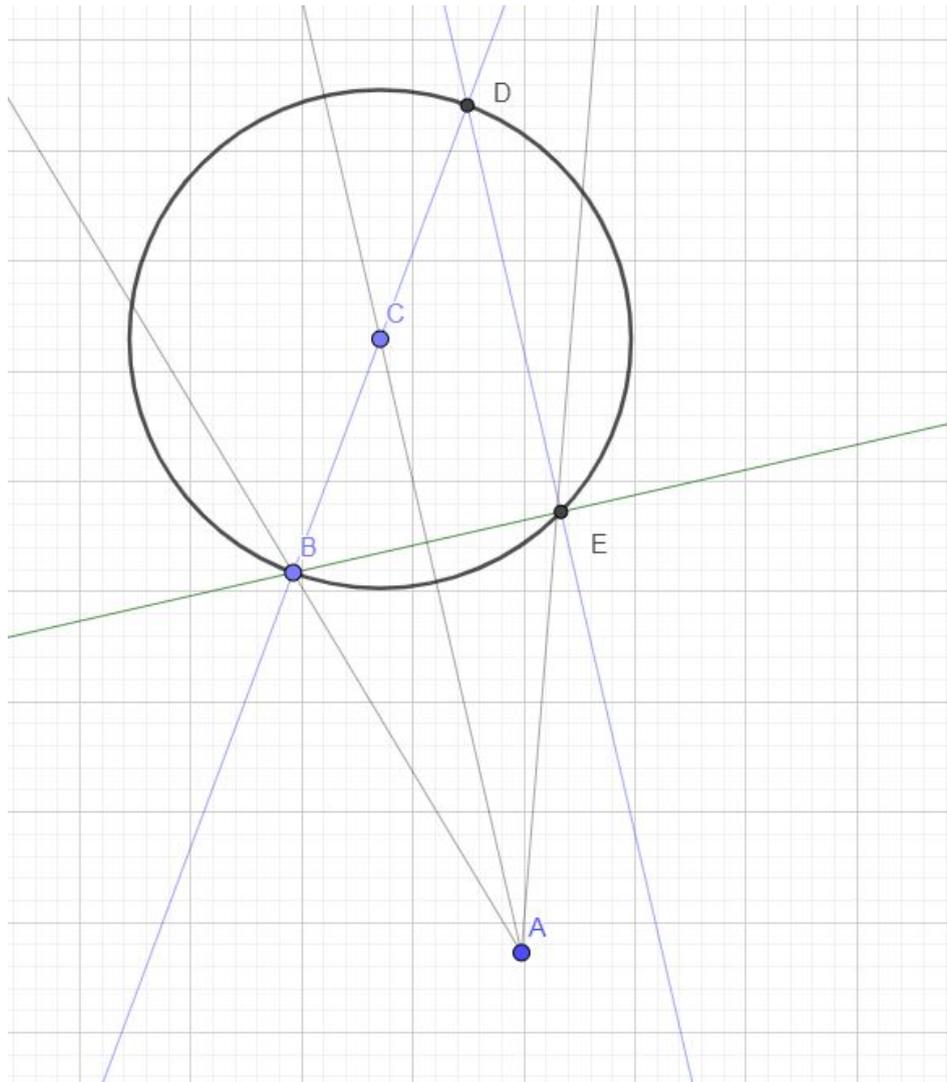
Just like before, we still don't know how far our target is, but we can repeat the exact same steps we took before (predict the fourth bearing - the red line, then move, and triangulate) with the same result. Note that we can obtain the same result if we draw our first point on bearing 3, and do the same procedure on the opposite side (find the mirrored point, and draw the parallel to bearing 2, intersecting bearing 1). We will come back to this in a moment.

The generalized four bearings method

Now for the real magic trick! Can we determine target course, speed and distance by taking four bearings *while on the move*? The answer is yes, but how we get there is a little more complicated. This is where I had the most trouble wrapping my head around the geometrical solution, and I'll present a slightly modified approach with a few more examples to help you see what's going on.

Now that we got the hang of bearings, the key idea to understanding what follows is that, *no matter what*, if the bearings are accurate they tell us that the target is *somewhere on that line*

at the time the bearing was taken. In other words, a bearing can be thought of as all the possible positions of a target at a given time. Hang on to that idea!

Now let's take a look at a "best case" situation, where we have taken three bearings on a target, while moving in a straight line, and they look like this:
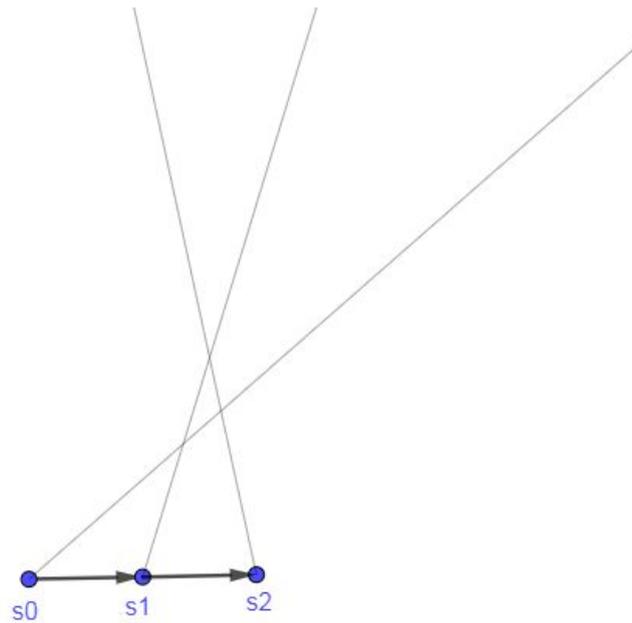
Where s0, s1 and s2 are our three positions. Note that while our speed is constant in this case, it doesn't have to be (and as we will see, neither does our course!), but we still have to take bearings at equal time intervals. Depending on the situation, this might be anywhere from three to thirty minutes (or more).

Now let's focus on that little triangle formed by our three bearing lines:
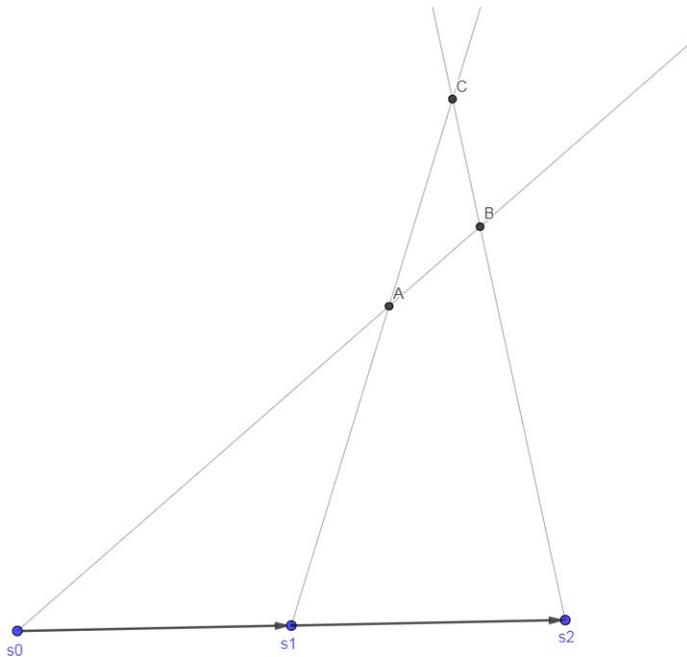
Here we will call A the intersection of bearings 1 and 2; B the intersection of bearings 1 and 3; and C the intersection of bearings 2 and 3.

Now, to see what we need to do, we have to think a little outside the box, and ask ourselves: what do we have to do in this case to find the "red line" from before? Or in other words, how can we predict the next bearing, if we don't even know the course? Well, the red line is just like any other line, so we only need *two points* to find it. If we could somehow find two points, from our drawings, that we *know* to be possible positions of our target *five minutes from when we took our third bearing*, then if we join them with a line, we will have *all* possible positions of our target after five minutes! Then we can simply do the same thing we did before, taking a fourth bearing, and triangulate. Let me show you what I mean, by spending some time reasoning on a few "absurd" situations. Just bear with me.

So, we have our three bearings, just like before. Now we get to the absurd part: what if we knew the target was traveling *exactly* along bearing 1? Most likely it wasn't, but if it was, when we took bearing 2 and 3, we would know exactly where the target has been, and its course and speed. And by the same reasoning, if we pretend that the target was actually *moving along bearing three* the whole time, we can use bearings 1 and 2 to know exactly where the target has been.
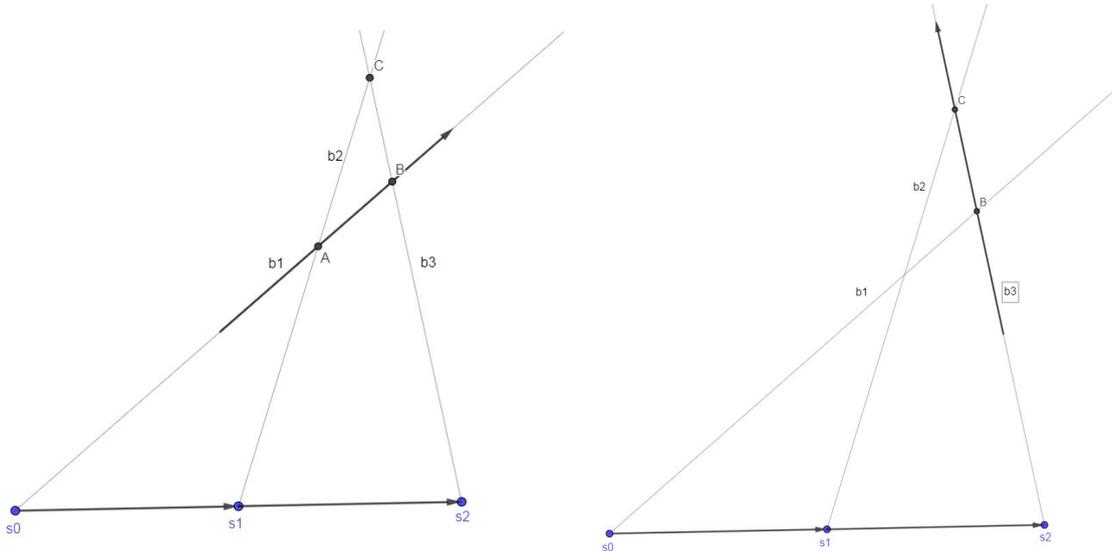

Do you see where I'm going? With these two "absurd" cases, we can find two *possible* positions for our target, five minutes from now. It's very unlikely that it was actually traveling exactly along bearing 1 or 3, but it *could have*. And as we discussed before, this is all we need to draw a line, and you'll have to trust me that that line will represent *all the possible positions* of our target, five minutes from now. Once again, we went from a "big infinity" of possible solutions, to a much more manageable *line*. So how do we find those two points?

If you remember our previous static example, once we determined course, we simply drew the red line by extending one possible target track by the same distance traveled in the previous five minutes, so that's exactly what we will do now:
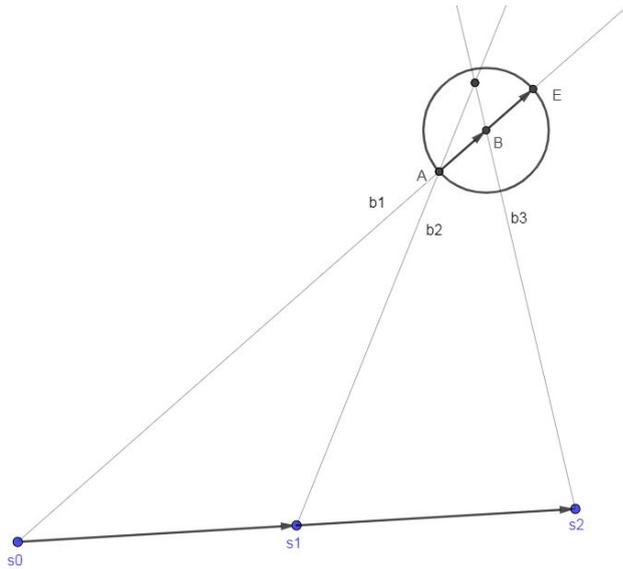
Simply draw a circle, centered on B (intersection of bearing 3 and 1) with radius BA; this is the simplest way to "copy" the AB segment forward, just like we did before. E is then our first possible position for the target, and the first point for constructing our red line.

Now we will do the same on bearing 3, with a small difference: we need to copy the BC segment along b3 *twice*, because in our "absurd" case, BC would be the distance that the target traveled from 10 to 5 minutes ago. So by protracting (*copying over*) BC the first time, we find where the target would be at the time we took bearing 3, and by doing it one more time, we will find what we need: point F, the second possible position of the target, five minutes from now (note that I've hidden the other circle and point E for clarity).
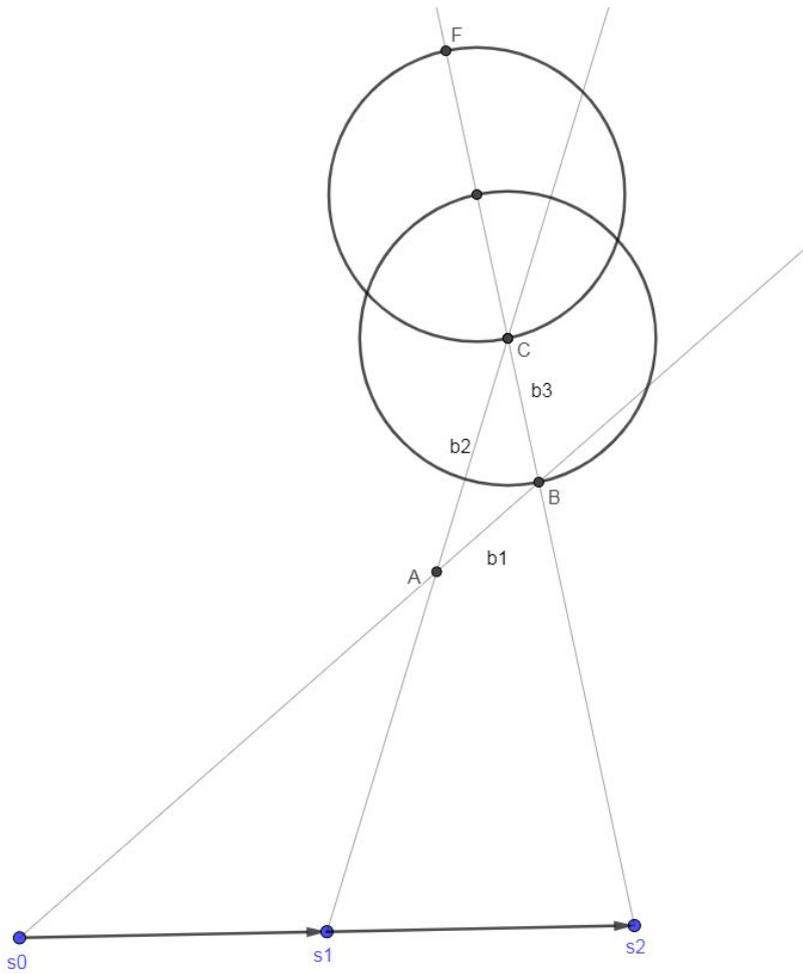
And if you've been following, you will now what's next: join points E and F with a red line, our predicted next bearing.
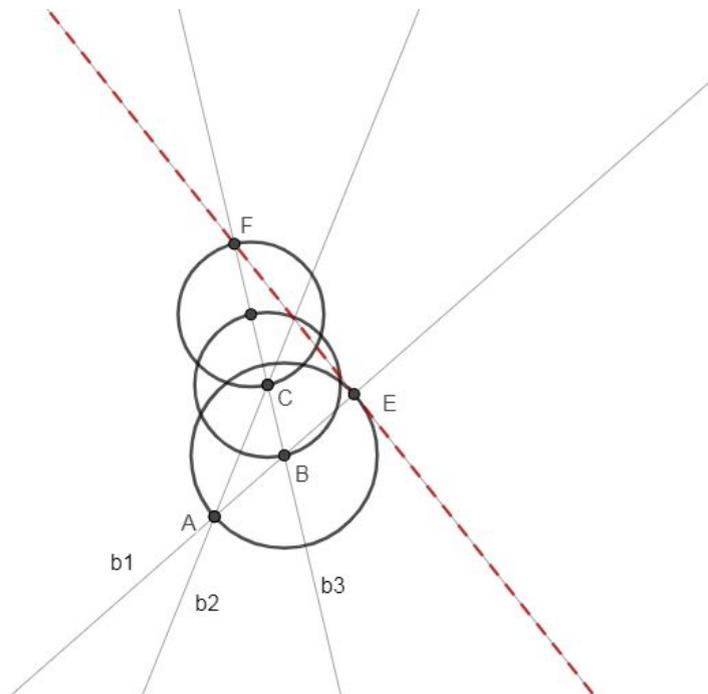
Yes, the drawing gets a little thick, but now we only need two little extra steps and we are done! We have our predicted bearing, but when the next time interval has passed, we will take one more bearing to triangulate. And here is the tricky part: if we have been moving at constant speed and bearing, now we absolutely need to change course, speed, or preferably both. If we keep running like we did, the next bearing we take *will coincide with the red line*, giving us no extra information, and preventing us from triangulating.

Just like before, the question is, which way to go to get the best solution? The answer is, *as far as possible from the red line*. The best result then would be either to double back on a perpendicular course to the red line from point s2, or to increase speed dramatically to pass the red line well before the next time interval runs out, but that might lead us away from the target. With experience you should be able to make an educated guess, but for the moment, let's just pick one to illustrate the method.
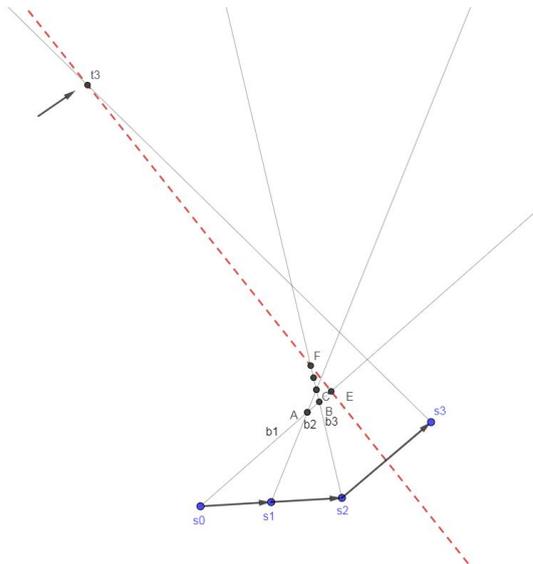

In this case we decided to cross the red line perpendicularly at max speed, which allowed us to take bearing 4 from point s3, which intersects the red line in t3, *the actual position of the target*. Wonderful, isn't it?

An important distinction: unlike the three bearings method before, we found the position *before* we found course. And this is where the second, alternative method for course estimation we saw earlier comes into play. Now that we know the target position, we will resolve its course by applying that simple method. So, from t3, we choose a random point (G in the picture below) on bearing 3 (why 3? Because bearing 4 is "now", and 3 is "one bearing ago"), and mirror t3 on the other side of the circle (point H) by drawing a line through t3 and G:
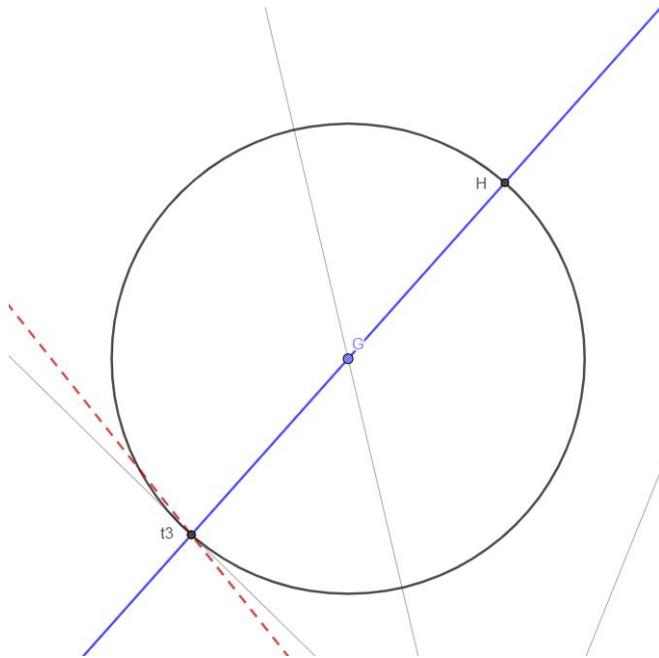
Now, like we did earlier, we draw a line parallel to bearing 3, through the new point H:
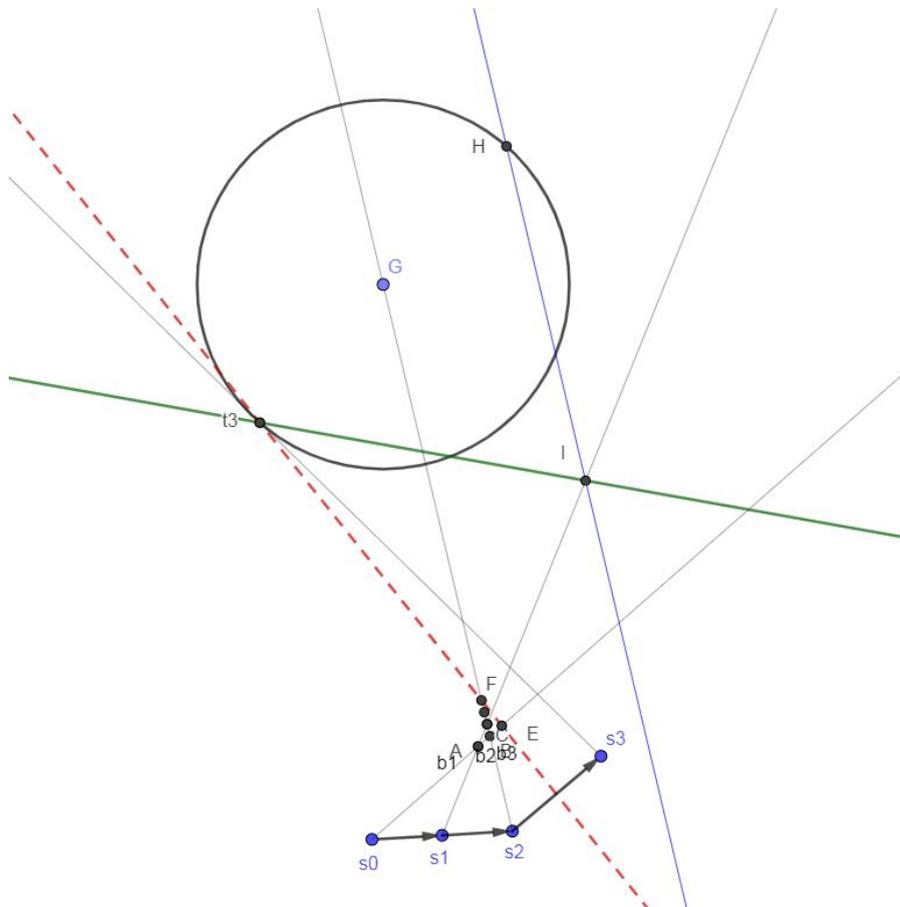
Thus finding point I on bearing 2 (which is "two bearings ago"). The green line is, as usual, the exact track of the target. Now all we need to do is measure one of the green segments between any of the two bearings, calculate the speed as before, and we have finished our job, as we know the exact position, course, and speed of our target, and we can work on our intercept.

And just to drive the point home, I'll show the *true* positions of our target, at the various times, that I used for our example (the blue points t0, t1, t2, and t3); the black t32 point is the position we derived with our construction:
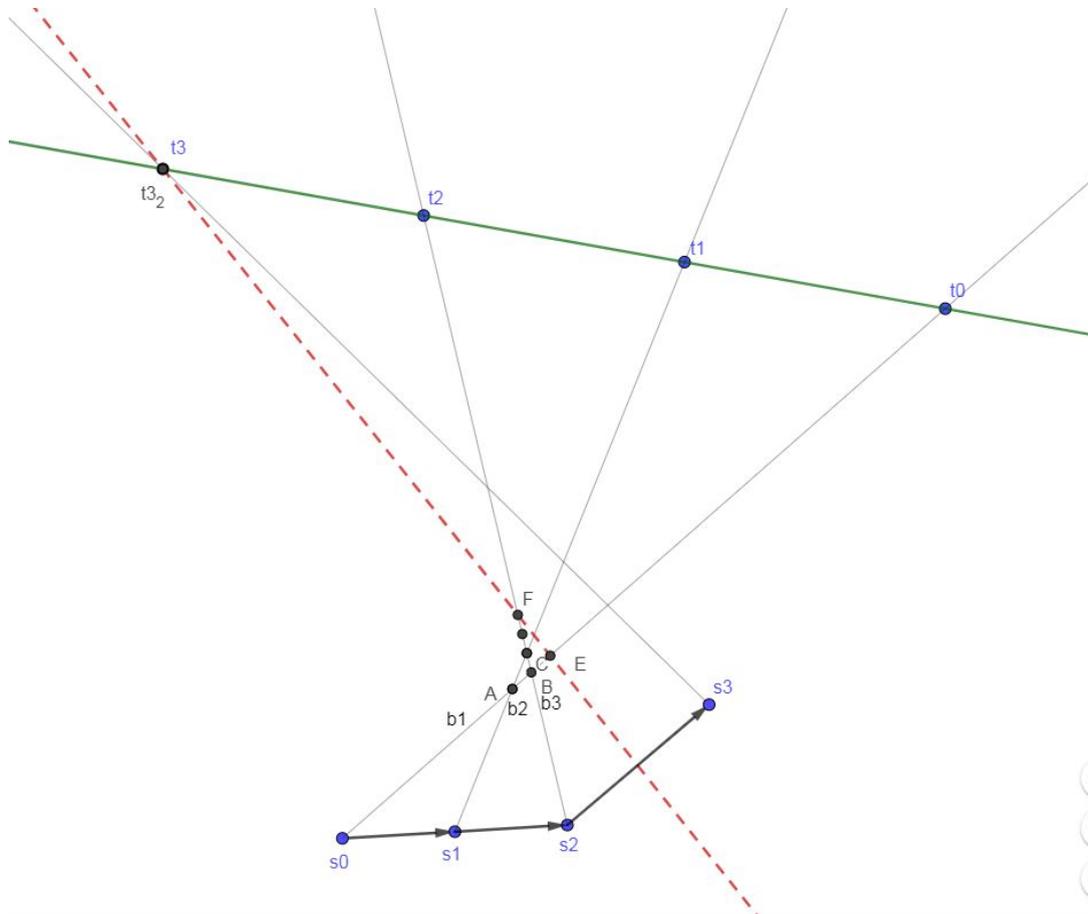

As you can see, it lines up perfectly, of course thanks to the fact that I'm using a computer and precise equations here, and all my bearings are perfectly accurate, so there's no room for errors, but I hope it will convince you of the result.

In the beginning it will look like a jumbled mess of lines and circles, but I guarantee, once you have done it a few times you will immediately see which side of the triangle you should protract and why, and you'll be able to do all this in a minute or less.

# But what if i have no magic triangle between my bearings?
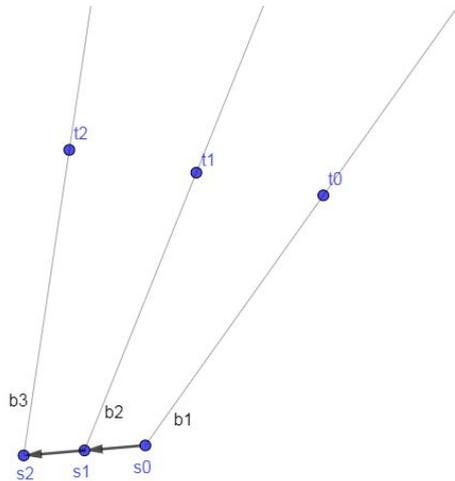
When I was first reading the original paper, I almost immediately wondered, what if my bearings are fanning out with no triangle to work with? This happens when you are running on a similar course to target, and at slower speed. So where do I find the segments to protract? The answer is, on the other side of my track! If we draw our bearings as lines, as opposed to rays from our ship, you'll see what I mean.
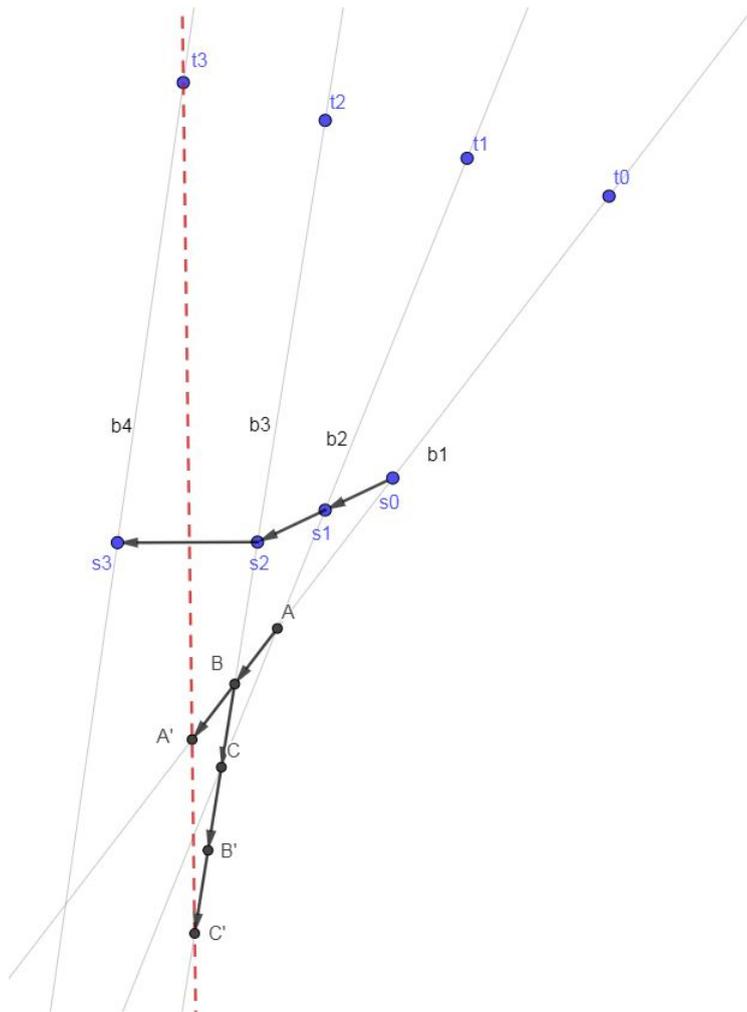
The triangle is still there, and as long as we protract the right segments in the right directions (just think of which bearing was recorded first, and you'll know), we still get a red line, and the fourth bearing will still intersect it at the right place. In this example, AB protracts to BA', and BC protracts to CB' first, and then B'C' (I haven't drawn the circles for clarity but you should get the hang of it by now). A'C' is our red line, and the blue points t0, t1, etc, still show the actual target position.

For completeness, here is another situation, where the contact is actually between you and *the triangle* (if the target is slower than you and moving in the same general direction):
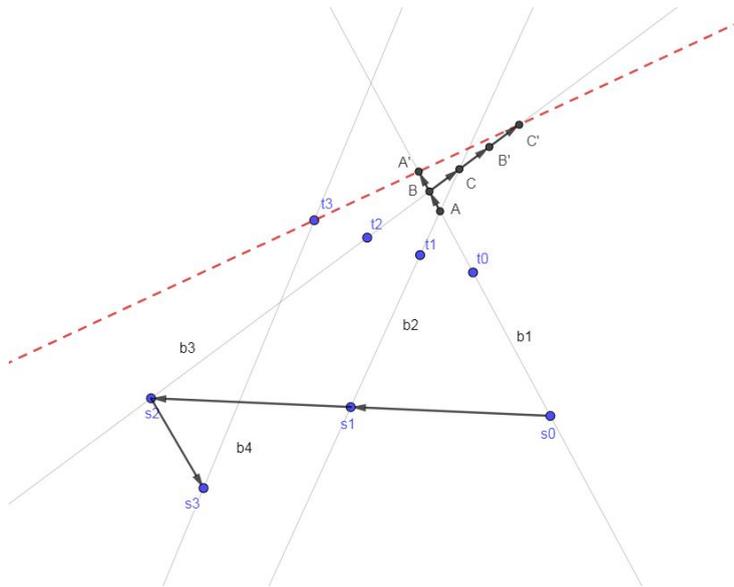
And just for fun, I'm doing a rather radical change of course for the last leg, which also works well for triangulation (remember, we want to be as far from the red line as we can).

Finally, remember: we don't have to move at constant speed at all! We can do all sorts of wonky things in between, but as long as we keep logging bearings with the same time intervals, we will be fine:
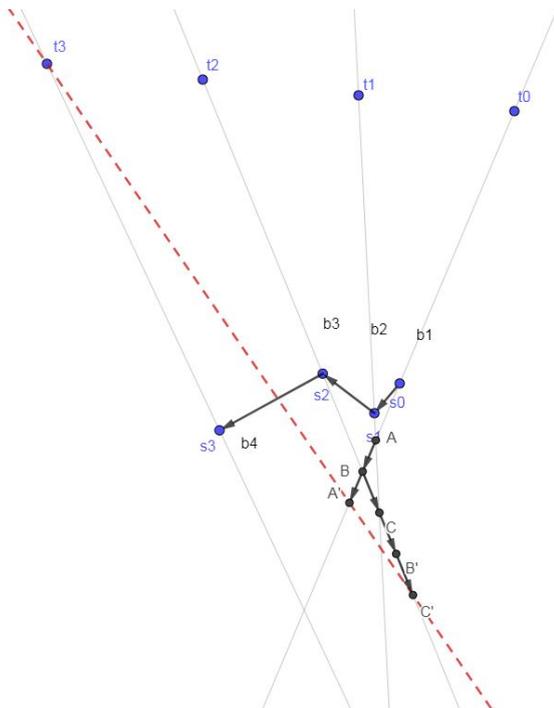
This is a little weird and extreme, and if you play *too much* with your course and speed, you will get weird triangles that cross your path and make your drawings confusing, so take this with a grain of salt and keep your path somewhat regular!

The last special case

There is one last interesting case, in which there is no triangle at all, even if we are moving. This happens when we are moving at constant speed, and on the same exact course (or the exact opposite) as our target:
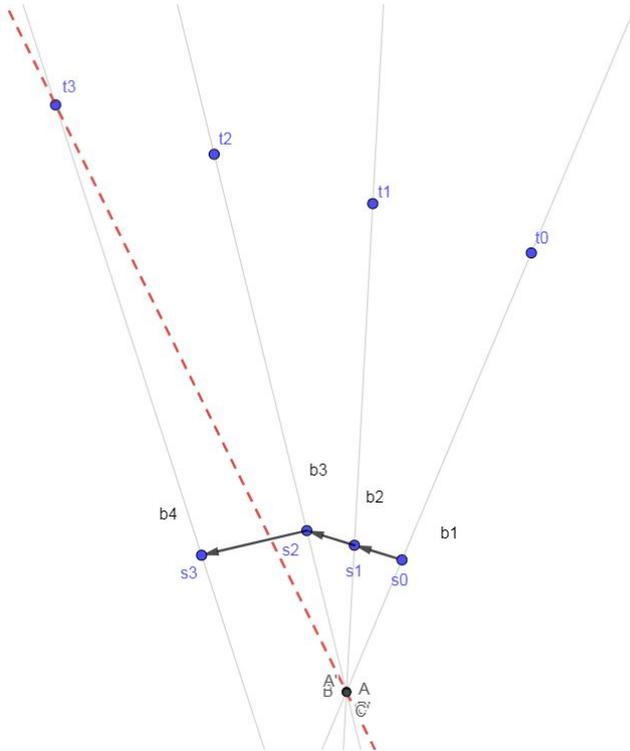
As you can see, in this case my tool still finds a perfect red line, but you would have a really hard time doing this on a chart! Either way, this case is simply a special case of our stationary three bearings method. We can apply the same approach (choose a random point on bearing 2, draw the parallels of b1 and b3 through it, and join the points to find course, then find the red line, and triangulate); this is because in this case, it's as if there was a "virtual sub" taking bearings in the point where all the real bearings converge, on the opposite side of the target.

I'm including this more like an interesting example, than an actual tool, because it's very unlikely that you will find this in practice. But if you do somehow end up in this situation, you are in luck, because you already know the target course and you can use the simple approach of the static case. Food for thought.

I could go on with examples and interesting properties, but I think for now this is where I will conclude my not so short rambling on bearings and geometry.

Good hunting!