The four bearings method (v2)

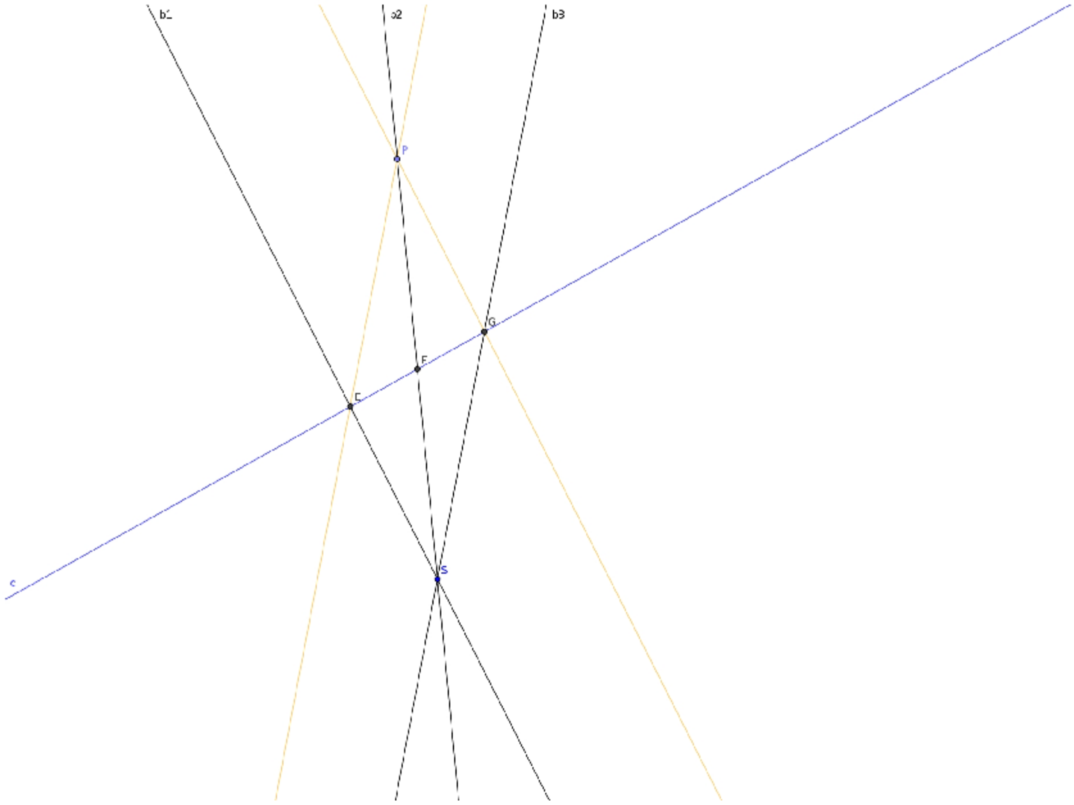
Most of the hunting in a submarine consists in collecting information about the target in order to fix its distance, course and speed, so to be able to be in the right place at the right time and find a firing solution. The nature of the available data depends on the circumstances, and once in a while we will find ourselves with only bearings to work with. For instance, when we can only use sonar.

Four bearings is enough to deduce all we need to know about the target (except draft, but that is marginal) and with a few easy graphical procedures in the chart it can be done fast and accurately.

The way it was done in real life was using a maneuvering board. We don't have this in SH, but the geometry implied is absolutely equivalent when done in the chart.

Before we start with computing the situation of target with only four bearings lets have a look at a few simplified problems. In what follows, we assume target moving straight and at constant speed, and bearings taken to it at equal intervals.

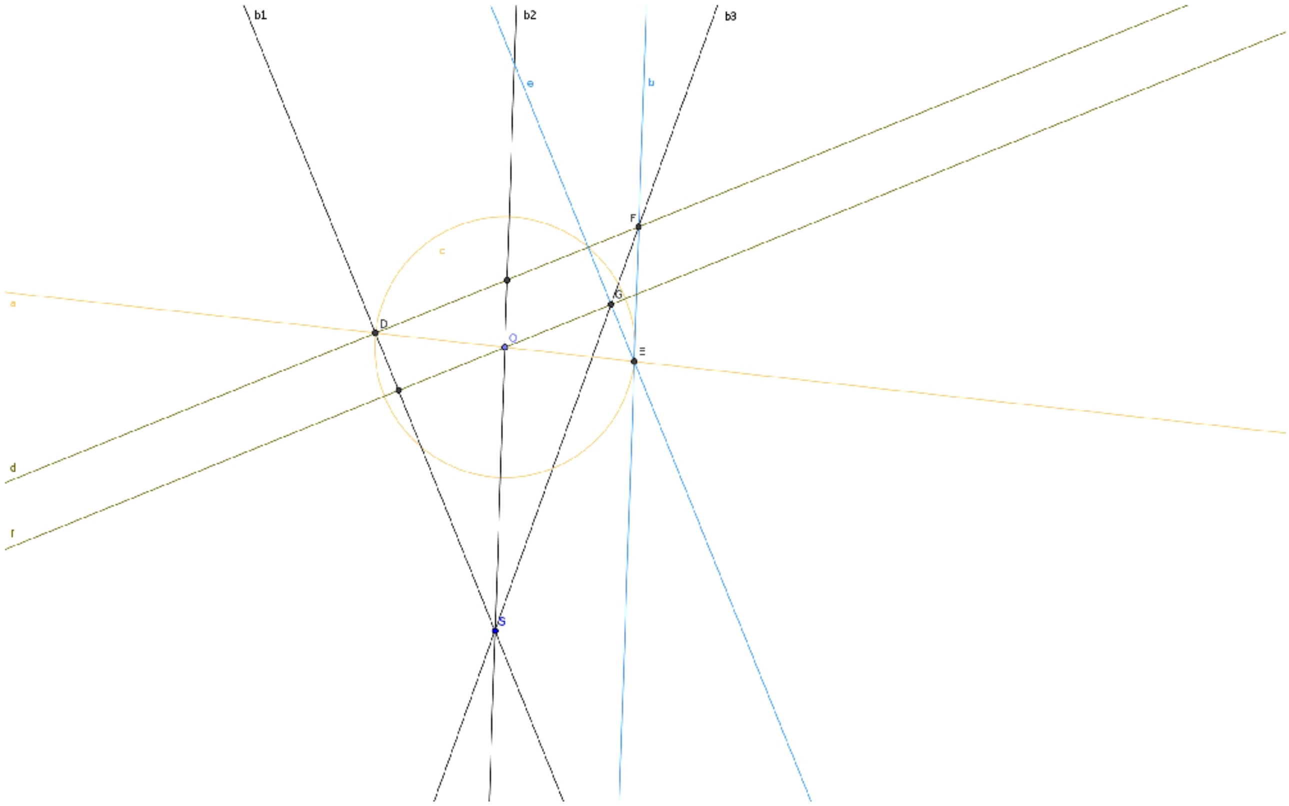
1. Submarine static. Given three bearings compute course.



Choose an arbitrary point **P** along the second bearing and draw parallels to the other two through it. Those cut the first and second bearing in points **E** and **G**, which determine the target's course.

2. Submarine static. Given three bearings compute course. (Alternative)

This procedure is a bit more complex, but it will also be of use when the submarine is moving, so we need to know it.

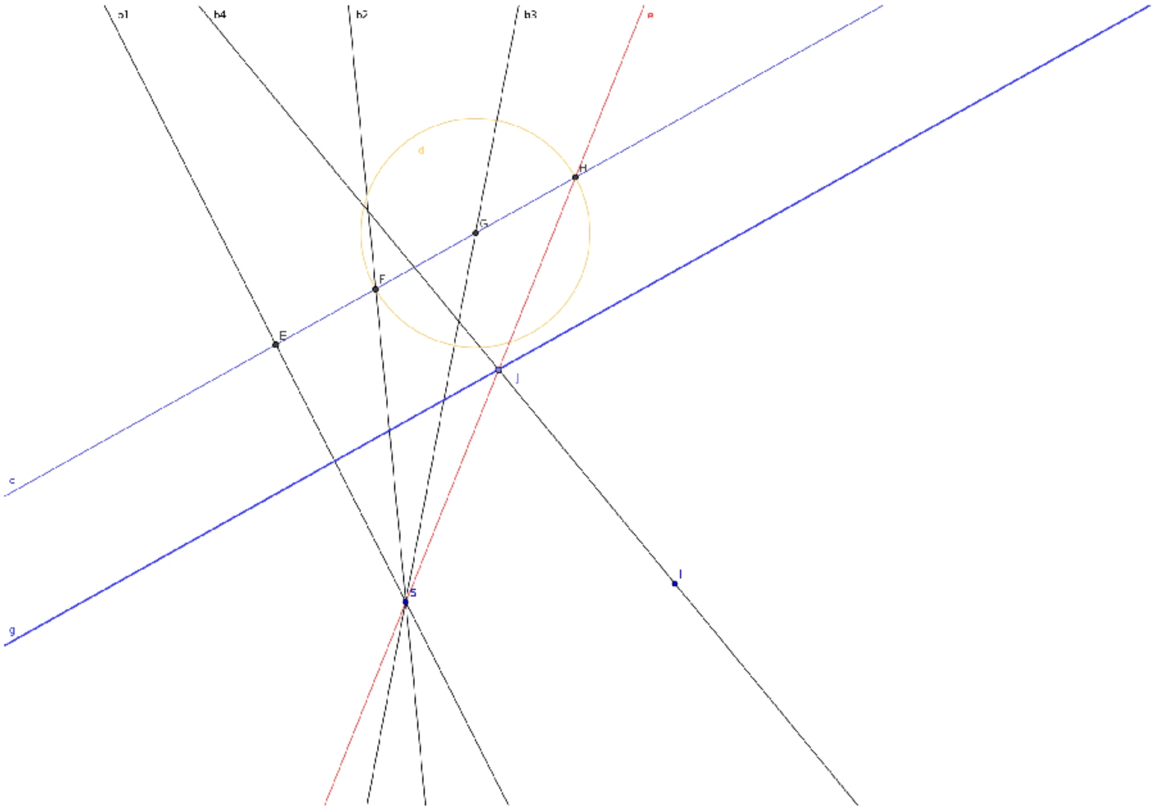


Choose two arbitrary points **D** and **Q** in bearings **b1** and **b2**, respectively. Draw the line **a** and find the point **E**, that mirrors **D** through **Q.**

Now we must choose which point we want to keep as part of our solution. If we want **D**, in the first bearing, we'll draw a parallel to **b2** through **E.** If we want **Q**, the parallel will be to **b1**. In either case, the parallel will cut **b3** at a point which, joined with the one we wanted to keep, will give us the course.

Observe that the courses thus obtained are parallel, but we chose where to draw them so if we knew the distance along any of the bearings we would have course, position and speed of target. This is also true if the submarine is moving, but if we don't have any distance we don't have course either in this case, because the possible courses won't be parallel and will depend on the initial choice.

3. Three bearings from a fixed point and triangulation.

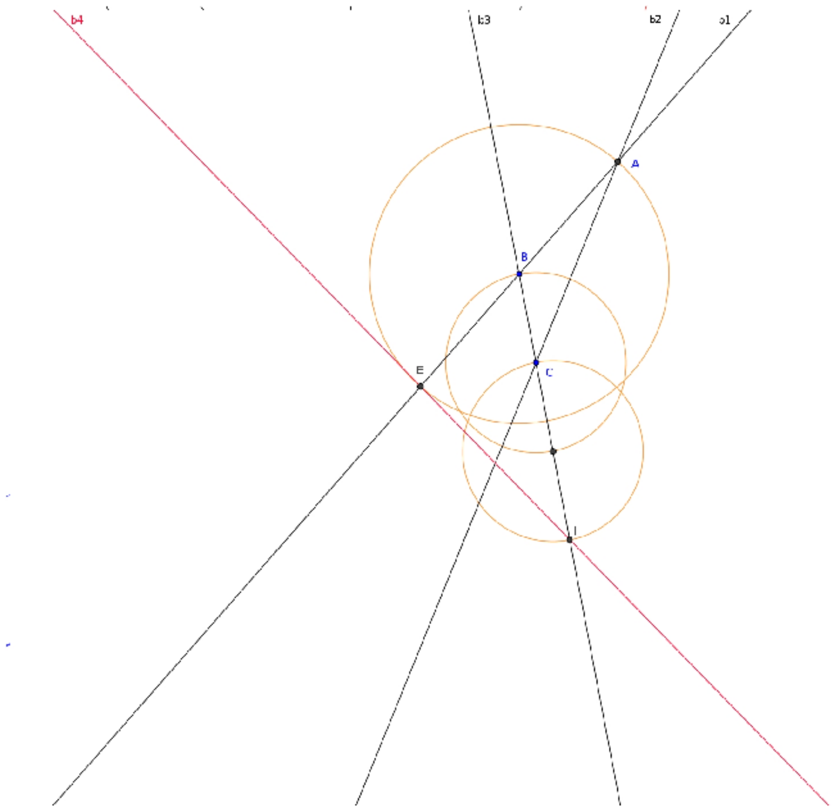


We have computed a course **c** from our fixed position **S** and so we have been able to draw an expected bearing (in red) for the next time interval (by protracting **FG** to **GH**). If we move our sub to **I**, and take a bearing **b4**, its intersection with the expected bearing gives us the true position of the target, and, since the course is parallel to the one we first estimated, we also have its speed from the length of the segments determined by the bearings.

4. The four bearings method.

In general, our vessel is moving while taking bearings. Those bearings don't meet at the same point like above; they form a triangle **ABC.** To compute an expected fourth bearing we could draw two possible courses using the procedures depicted in the case 2, protract the travelled distances along them and join the resulting points. This is how it was described in the first version of this document, but there is an easier and faster way.

It is consistent with the three taken bearings and expected one we try to compute that the target is traveling along any of them. If the target were traveling along **b1**, then **b2, b3** and **b4** have to cut **b1** so to make two equal segments in it. If the target were traveling along **b3**, the segment that **b1** and **b2** define on it should be half the length of the one defined by **b2** and **b4** (because it is a double interval of time). The analogous can be reasoned for **b2.** So we have a very easy way to compute a fourth expected bearing (the red line in version 1 of this document):

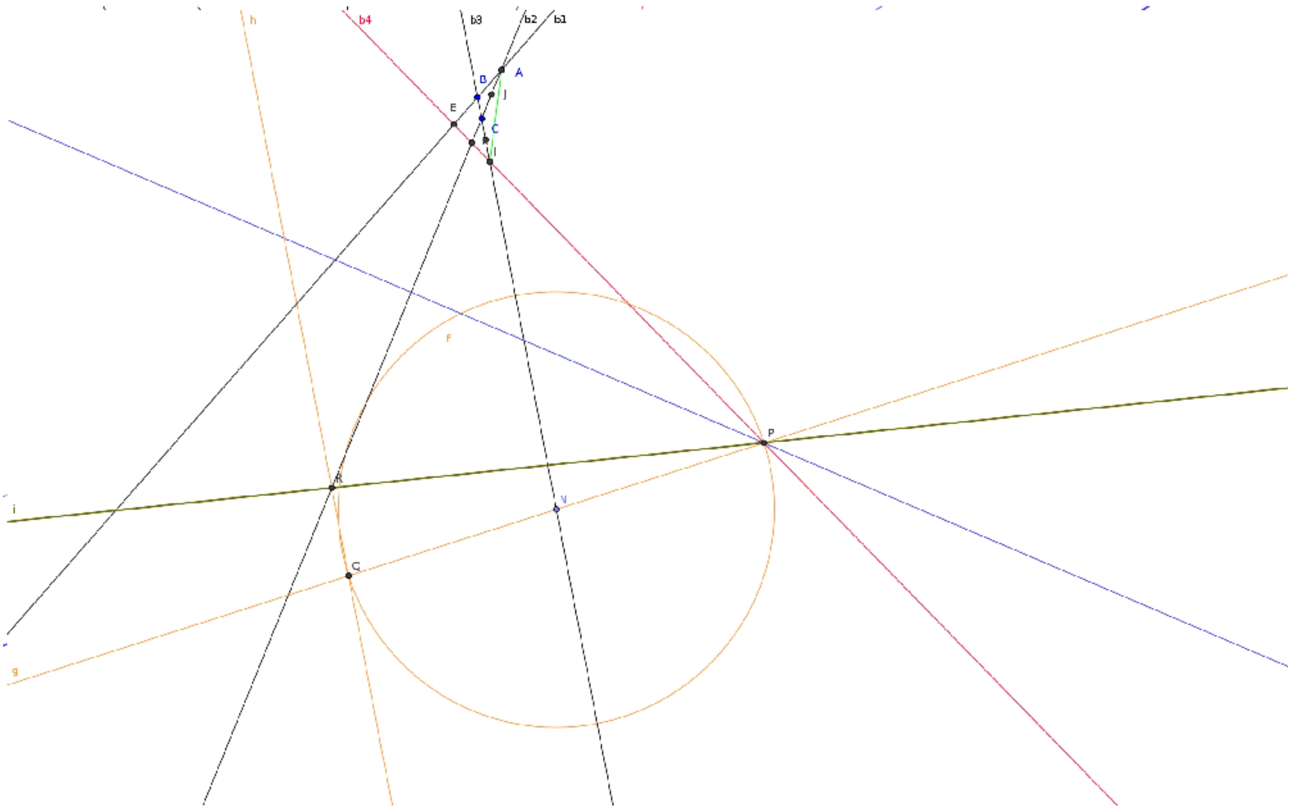


Protract the segment **AB** along **b1** to obtain **E.**

Protract *twice* **BC** along **b3** to obtain **I.**

The line defined by **E** and **I** (in red) is the expected bearing.

If we have been traveling at constant speed and course while collecting data and keep doing so, the next bearing we take will be that red line, because our own course is one of the possible solutions (thanks, Makman) so, like in section 3, we need to triangulate, that is: go where our next bearing cuts the expected one as perpendicularly as we can. The intersection point will be the true position of the target, and we can use the method from section 2 to compute its course (and therefore its speed) from that point:



Here the blue line is the fourth (real, not estimated) bearing we have taken, finding the position of the target at **P.** From there, we compute the true course of the target (in green) following the method from section 2 and we're done.

Practical considerations

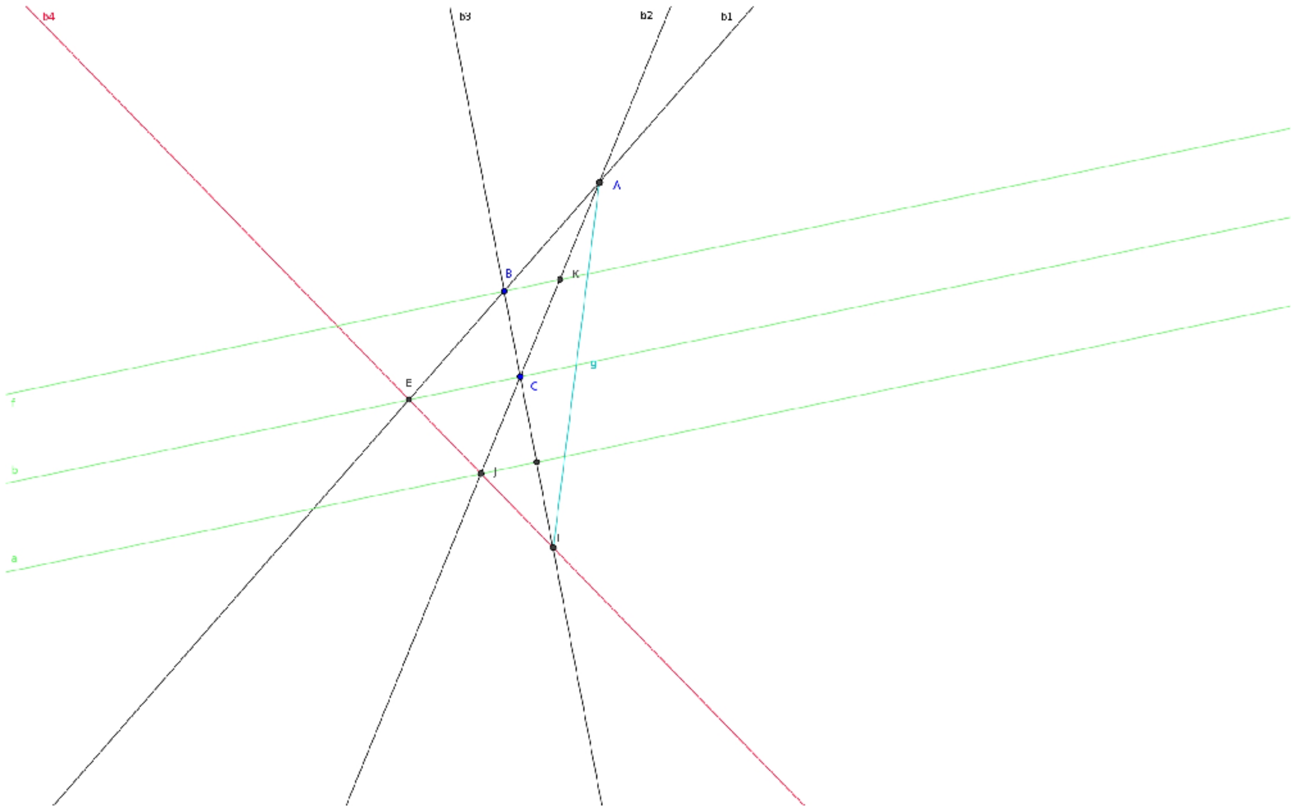
Change of course or speed for triangulation can be made at any time between **b1** and **b4.** Changing only speed won't help much. The objective of moving away from a straight course at constant speed is to obtain a bearing that makes an angle as wide as possible with the one we would obtain otherwise, and that calls for a drastic change of course for a good while before taking some of the following bearings.

You have to be aware of the possibility that the solution is at the other side of the triangle, in which case the target would be traveling in a opposite course to ours. To assess this, pay attention to the remarks of your sonar man ("moving away") or any other data that may allow you to discard or confirm that possibility, like the other side being too far away to be hearing anything, etc.

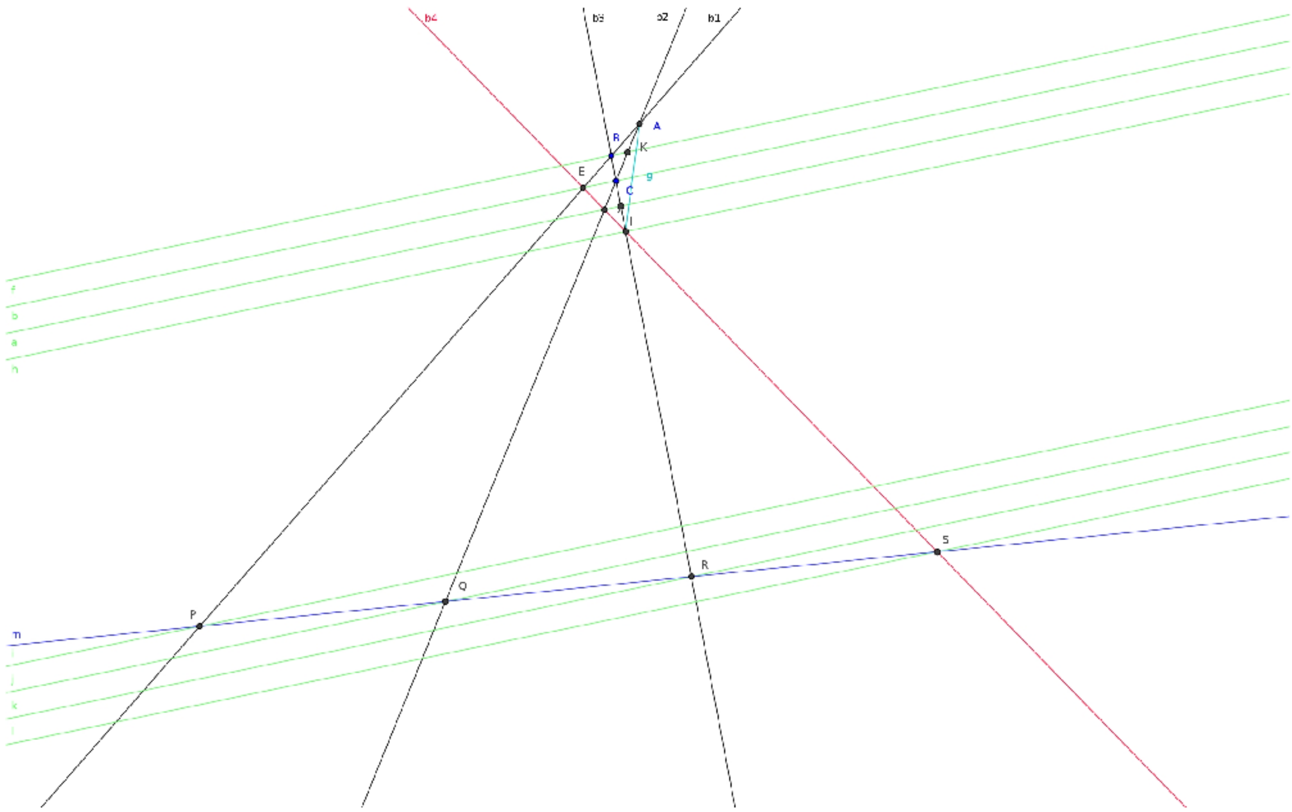
If you have a rough idea of the speed of the target you can improve your situation awareness by estimating a possible course for it after three bearings. Actually, if you *know* the speed, you can compute a solution without waiting for the fourth. To do this, let me first call your attention to a interesting property of the initial geometric construction we have been using.

Those familiar with basic maths may already have noticed that **b2** and **b3** define the medians of the triangle **AEI** that fall in the sides defined by **b1** and **b4.**

Lets have a look at the third median, along **EC**:



The lines through **BK** and the analogous through **J** are obviously parallel and equally spaced. So are the parallels through **A** and **I.** They divide all four bearings (remember that the fourth is calculated from the other three) in equal segments and, if taken anywhere, will give us a possible solution course, like this:



We can take advantage of this property to solve the next problem:

5. Given three bearings and speed of target, compute its position and course.

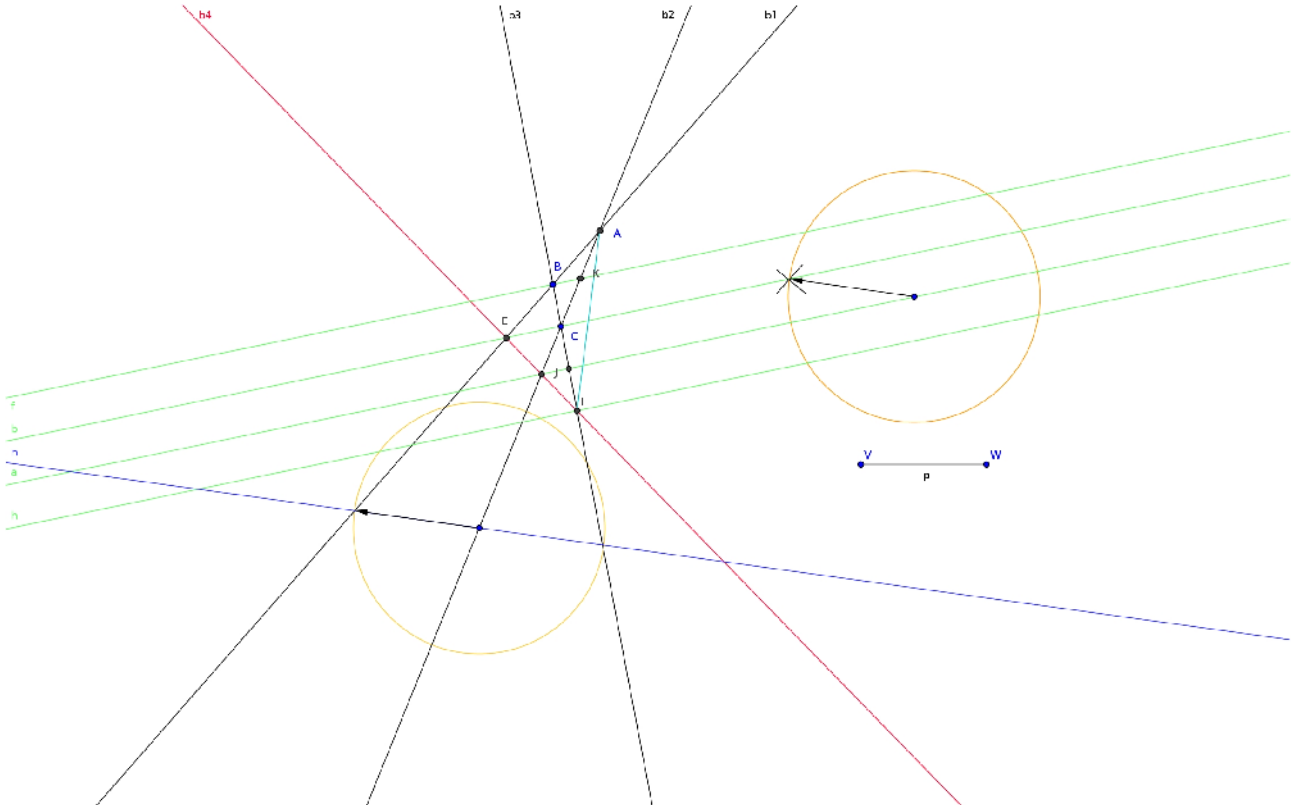
Since it may not be obvious how to handle this with the tools on board, this time we are going to use them explicitly.

Although they are drawn in the figure, we don't need the four green parallels, only two of them, contiguous.

So we know speed of target. Then we know the distance traveled by it for the time interval between bearings.

Draw a segment corresponding to that distance **VW** and a circle (compass tool) with that radius. Take the circle to the green lines, so it centers in one of them. Mark the intersection of the circle with the next green line. There are two such points so choose the plausible one; the other corresponds to the solution at the other side of the triangle. Take the radius to point to the mark. The radius now is already giving us the true course of the target. If we have chosen the wrong one, we'll know in a minute.

Now drag the circle to the bearings so its center touches one and the tip of the radius touches the next.



The target is following the blue line. If the blue line does not define equal segments at the bearings, go back and choose the other point of intersection. It will work this time.

This procedure could be use twice to compute bounding courses for corresponding bounding speeds. If we know that the target moves between 5 and 7 knots, two *blues lines* like the one above would enclose all the possible courses and positions for it.

Conclusion

The advantage of this methods is that collection of data is not forcing us to remain static or move at constant speed and course. We can maneuver quite freely and work in getting to an advantageous position while gathering information. As usual, knowledge means freedom. Although some of the methods described may feel daunting at first sight, once they have been put to practice a couple of times they are easy and fast to apply. If you, like me, are all for full realism, I think that you'll be satisfied with these tools in your bag. I hope that you have so much fun using them as I had writing about them.

I'll be happy to answer any questions and discuss the maths involved. Meet me at *Subsim Forums.*

Anyone willing to improve on my English or any other aspect of this tutorial is welcome. PM me and I'll send you an editable version of the document to work with.

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Good hunting.

Kuikueg